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Residual Stress Measurement Techniques: SET-Cos-Alpha, SET-Sin$^2\psi$, MET-Sin$^2\psi$

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Outline

1. Theory of the different techniques
   - $\mathrm{Sin}^2\psi$ and $\mathrm{Cos}\alpha$
     - Multiple Exposure Technique (MET)
     - Single Exposure Technique (SET)
     - Stress under applied load
2. Simulation of SET- $\sin^2\psi$, MET- $\sin^2\psi$ and SET-$\mathrm{Cos}\alpha$
3. Stress measurements on samples and 4-point bend specimen
4. Results and discussions
5. Conclusions

Instrument evaluation and qualification

What is available?

1. Free-stress powder (normal and shear stress)
2. Shot peened high stress reference standards (normal stress, no shear stress)
3. Stress under uniaxial loading (relative change in stress)
4. Perform gauge R&R

Structure of an instrument

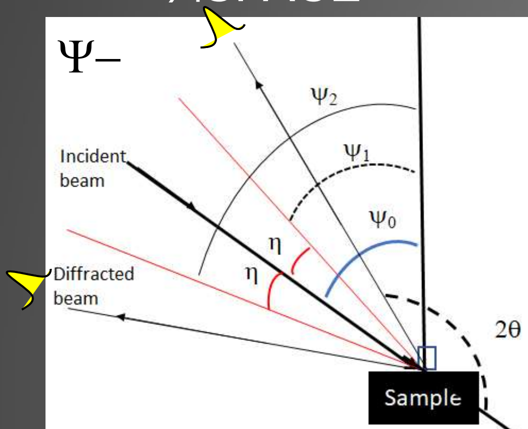
1. Goniometer geometry
2. Detector, X-ray source, optics
3. Software: peak location methods, **STRESS MODEL,** other options

History of stress measurement simplified

1. Two points using film or detector ➔ Normal stress σ_ϕ
2. Multiple points using detector (ψ+, $\sin^2\psi$) ➔ Normal stress σ_ϕ
3. Multiple points using detector (ψ+ and ψ-, $\sin^2\psi$) ➔ Normal stress σ_ϕ and Shear stress τ_ϕ (τ_{13}) (Standard documents)
4. Cosα using Debye ring ➔ Normal stress σ_ϕ

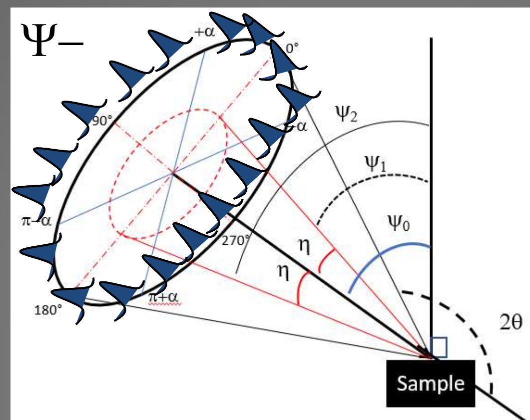
Measurement techniques

¼SPACE



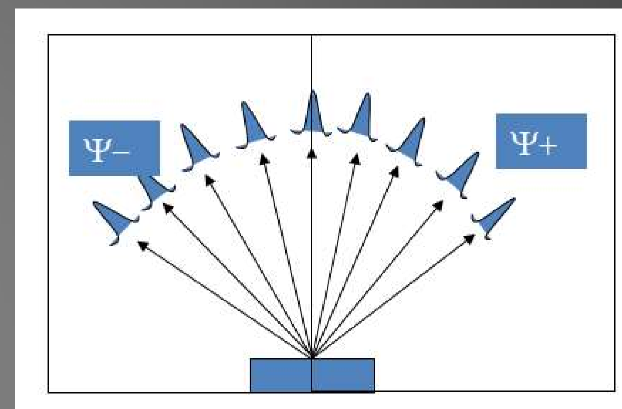
SET = 1 side tilt

¼SPACE



SET Cosα: 1 side tilt

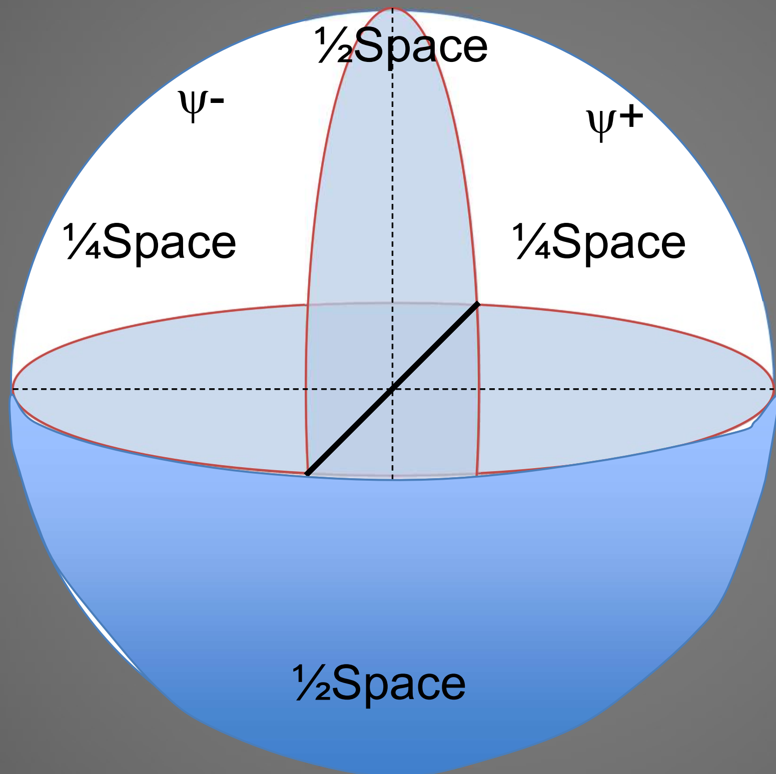
½SPACE



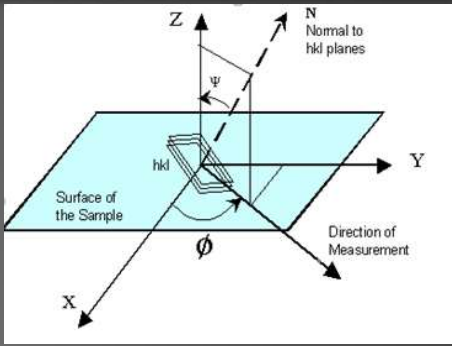
MET = 2 sides multiple tilts

ψ_0; $\psi_1=\psi_0+\eta$ and $\psi_2=\psi_0-\eta$, where $\eta=(\pi-2\Theta)/2$.

- SET-Sin$^2\psi$ uses 2 ψ-angles with a single tilt position.
- SET-Cosα uses multiple angles around the Debye ring and one tilt position.
- MET-Sin$^2\psi$ uses multiple angles distributed on 2 sides with ψ^+ and ψ^-.


½Space
ψ-
ψ+
¼Space
¼Space
½Space

$\text{Sin}^2\Psi$ technique - MET-$\text{Sin}^2\psi$ technique



- Strain: $\varepsilon_{\phi\psi} = \frac{D_{\phi\psi} - D_0}{D_0}$
- For triaxial stress state, $\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$, $\epsilon_{ij} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \varepsilon_{23} & \epsilon_{33} \end{pmatrix}$
- Calculate strains in the direction $n_i = \begin{pmatrix} sin\psi\ cos\phi \\ sin\psi\ sin\phi \\ cos\psi \end{pmatrix}$, $\varepsilon_{\phi\psi} = \varepsilon_{ij} n_i n_j$
- Hooke's law for isotropic materials : $\varepsilon_{ij} = \frac{1+\upsilon}{E}\sigma_{ij} - \frac{\upsilon}{E}(\sigma_{11} + \sigma_{22} + \sigma_{33})\delta_{ij}$
- MET-$\text{Sin}^2\psi$ equation is:

$$\varepsilon_{\phi\psi} = \frac{1+\upsilon}{E}(\sigma_{11} cos^2\phi + \sigma_{22} sin^2\phi + \sigma_{12} cos2\phi - \sigma_{33}) Sin^2\psi + \frac{1+\upsilon}{E}\sigma_{33} - \frac{\upsilon}{E}(\sigma_{11} + \sigma_{22} + \sigma_{33}) + \frac{1+\upsilon}{E}(\sigma_{13} cos\phi + \sigma_{23} sin\phi) Sin2\psi$$

- Both normal and shear stresses can be calculated when multiple tilts in both directions are selected (ψ>0 and ψ <0).
- *σ_{i3} =0, except (σ_{13}, σ_{23})≠0 because of the non-uniformity distribution of strain generated by the process applied*

- In case of a single direction measurement, assuming $\sigma_{33} = 0$ because of the shallow penetration of x-rays, the equation can be written as :

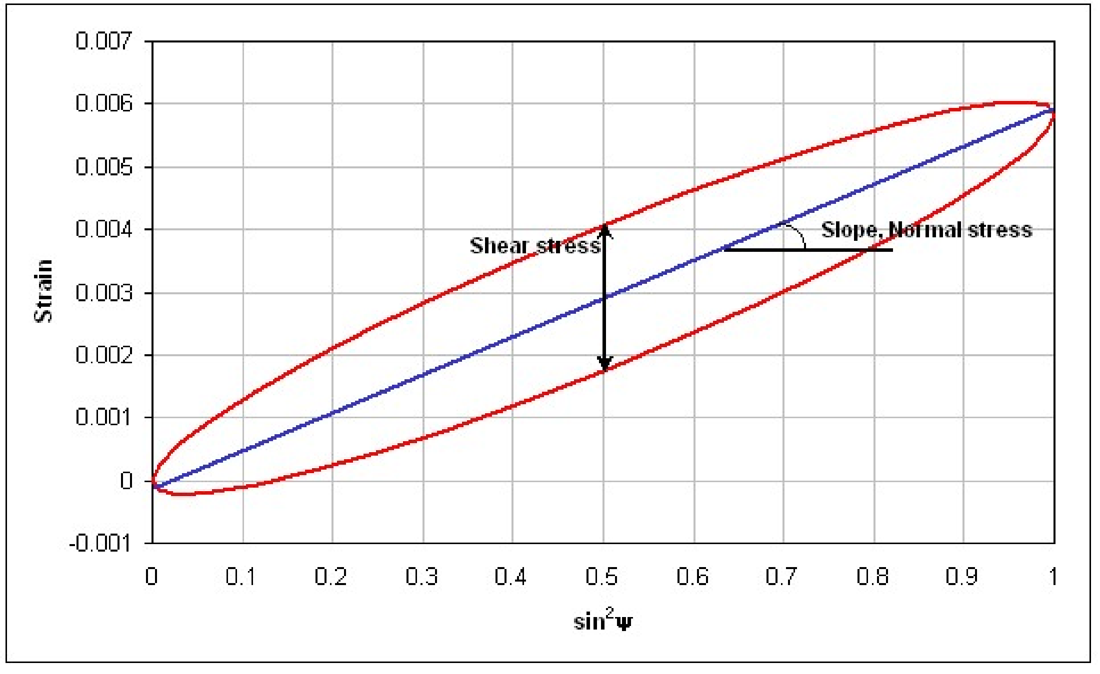
$$\varepsilon_{\phi\psi} = \frac{1+\nu}{E}(\sigma_\phi)\sin^2\psi + \frac{1+\nu}{E}(\tau_\phi)\sin 2\psi - \frac{\nu}{E}(\sigma_{11}+\sigma_{22})$$

$\frac{1+\nu}{E}=\frac{1}{2}S_2$ and $-\frac{\nu}{E}=S_1$

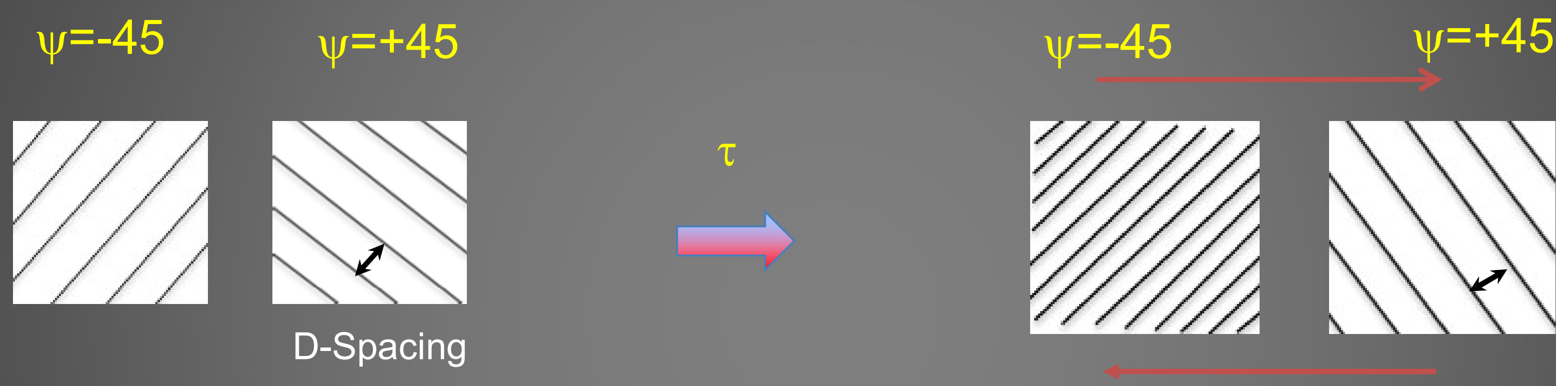
- Using elliptical regression:

$$\varepsilon_{\varphi\psi} = A\sin^2\psi + B\sin 2\psi + C$$

- This model is exclusively used in most stress software.



Change of D-spacing after applying a shear stress



When shear stresses can be measured:

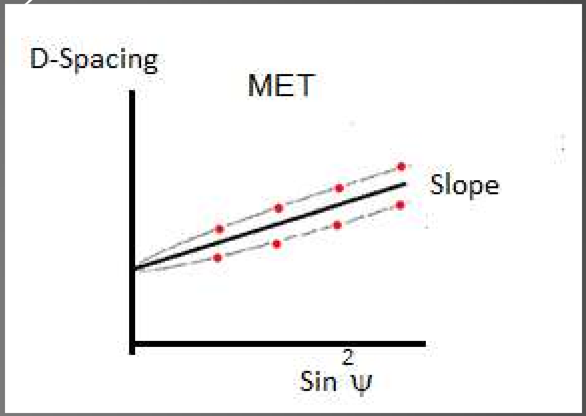
- Heterogeneous materials
- Non-uniform processes
- Curved surfaces
- Tilted surfaces

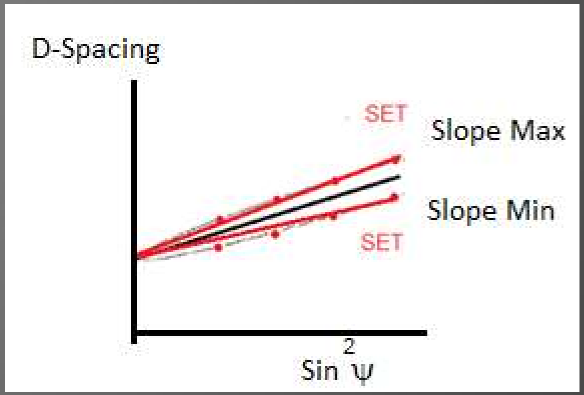
Single Exposure technique (SET)

- The $Sin^2\psi$ equation in this case can be reduced to the following form for $\phi=0$:

$\sigma_{11} = \frac{\Delta\varepsilon_{\psi}}{\Delta Sin^2\psi}\left(\frac{E}{1+\upsilon}\right)$ slope between 2 points.

Only Normal stress can be calculated.





Cos-Alpha

- For triaxial stress state, $\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$, $\epsilon_{ij} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \varepsilon_{23} & \epsilon_{33} \end{pmatrix}$
- Hooke's law for isotropic materials :

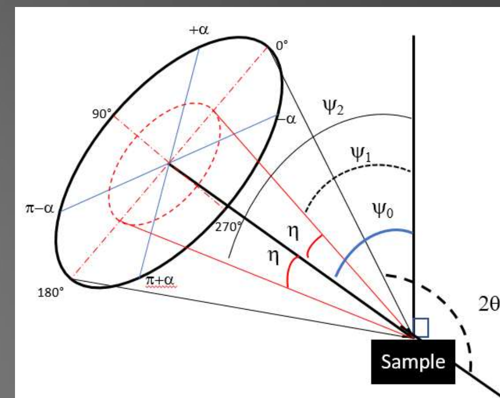
$$\varepsilon_{ij} = \frac{1+\upsilon}{E}\sigma_{ij} - \frac{\upsilon}{E}(\sigma_{11} + \sigma_{22} + \sigma_{33})\delta_{ij},$$

- Calculate strains in the direction (ψ_0, ϕ_0, α), $\varepsilon_{\varphi\psi} = \varepsilon_{ij} n_i n_j$

where $n_i = \begin{pmatrix} cos\eta\ sin\psi_0 + sin\eta\ cos\psi_0\ cos\alpha \\ cos\eta\ sin\psi_0 sin\phi_0 + sin\eta\ cos\psi_0\ sin\phi_0 cos\alpha + sin\eta\ cos\phi_0\ sin\alpha \\ cos\eta\ cos\psi_0 - sin\eta\ sin\psi_0 cos\alpha \end{pmatrix}$

- For ϕ_0=0:

$$n_i = \begin{pmatrix} cos\eta\ sin\psi_0 + sin\eta\ cos\psi_0\ cos\alpha \\ sin\eta\ sin\alpha \\ cos\eta\ cos\psi_0 - sin\eta\ sin\psi_0 cos\alpha \end{pmatrix}$$



- The Cos-Alpha equation is:

$$a_1 = \frac{1}{2}[(\varepsilon_\alpha - \varepsilon_{\pi+\alpha}) + (\varepsilon_{-\alpha} - \varepsilon_{\pi-\alpha})$$

$$a_2 = \frac{1}{2}[(\varepsilon_\alpha - \varepsilon_{\pi+\alpha}) - (\varepsilon_{-\alpha} - \varepsilon_{\pi-\alpha})]$$

ε_α, $\varepsilon_{\pi+\alpha}$, $\varepsilon_{-\alpha}$ and $\varepsilon_{\pi-\alpha}$ are the measured strains from the 4 sectors of the Debye ring.

$$a_1 = -\frac{1+\upsilon}{E}(\boxed{\sigma_{11}} sin2\psi_0 + \boxed{2\sigma_{13}} cos2\psi_0) sin2\eta. Cos\alpha$$

$$a_2 = 2\frac{1+\upsilon}{E}(\boxed{\sigma_{12}} sin\psi_0 + \boxed{2\sigma_{23}} cos\psi_0) sin2\eta. Sin\alpha$$

This is the true correct equation that should be considered.

It is clear from these equations that the components σ_{11} and σ_{13} cannot be separated and that σ_{11} and σ_{12} can be measured accurately only if σ_{13}=0 and σ_{23}=0.

If σ_{13}=0 and σ_{23}=0, (biaxial stress state), the previous equations are reduced to:

$$a_1 = -\frac{1+\upsilon}{E}(\sigma_{11}sin2\psi_0 + \cancel{2\sigma_{13}cos2\psi_0})sin2\eta.Cos\alpha$$

$$a_2 = 2\frac{1+\upsilon}{E}(\sigma_{12}sin\psi_0 + \cancel{2\sigma_{23}cos\psi_0})sin2\eta.Sin\alpha$$

and

$$(\sigma_{11}) = -\frac{E}{(1+\upsilon)sin2\psi_0 sin2\eta}\frac{\partial a_1}{\partial Cos\alpha}$$

$$(\sigma_{12}) = \frac{E}{2(1+\upsilon).si\quad{}_0.sin2\eta}\frac{\partial a_2}{\partial Sin\alpha}$$

σ_{11} and σ_{12} are calculated by plotting the quantities a_1=f(Cosα) and the quantities a_2=f(Sinα).

This what the Cosα instruments use in the software. Is this acceptable?

Applied loading:

- Under applied load, the stress equations become:

MET- Sin²ψ: $\varepsilon_{\varphi\psi} = \frac{1+\upsilon}{E}\left(\sigma_{11}^{A} + \sigma_{11}^{RS}\right)Sin^2\psi - \frac{\upsilon}{E}\left(\sigma_{11}^{A} + \sigma_{11}^{RS}\right)$

SET- Sin²ψ: $(\sigma_{11}^{A}+\sigma_{11}^{RS}) = \frac{E}{1+\upsilon}\frac{\partial \varepsilon_{\psi}}{\partial Sin^2\psi}$

SET-Cosα: $(\sigma_{11}^{A}+\sigma_{11}^{RS}) = -\frac{E}{(1+\upsilon)sin2\psi_0.sin2\eta}\frac{\partial a_1}{\partial Cos\alpha}$

σ_{11}^{A}: is the applied load.

σ_{11}^{RS}: is the residual stress measured before loading.

$(\sigma_{11}^{A}+\sigma_{11}^{RS})$: is the total stress measured during the experiment .

The slope of the measured stress versus applied stress is: $\mathrm{m} = \frac{\partial \mathrm{f}(\sigma_{11}^{\mathrm{A}}+\sigma_{11}^{\mathrm{RS}})}{\partial(\sigma_{11}^{\mathrm{A}})}$

1. Simulation:
   a) Residual stress at different inclination angle Ψ_0 , Ψ_0 was varied form 0° to 90°
   b) Simulation for 2 stress conditions:

$$\sigma_{ij}(MPa) = \begin{pmatrix} +500 & +100 & +100 \\ +100 & +500 & +100 \\ +100 & +100 & 0 \end{pmatrix} \text{ and } \sigma_{ij}(MPa) = \begin{pmatrix} +500 & +50 & +50 \\ +50 & +500 & +50 \\ +50 & +50 & 0 \end{pmatrix}$$

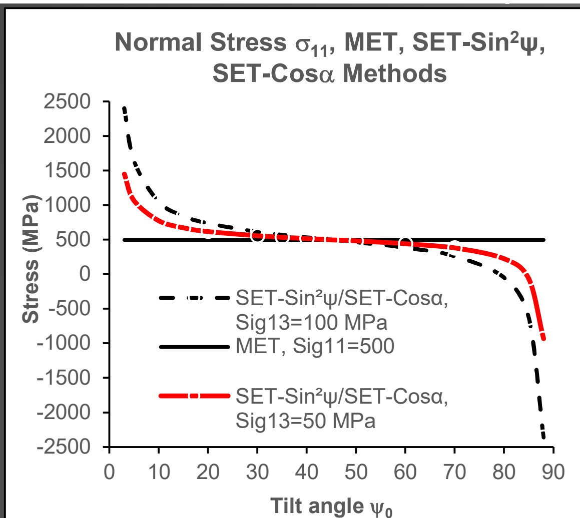
1. Experiment - Simulation

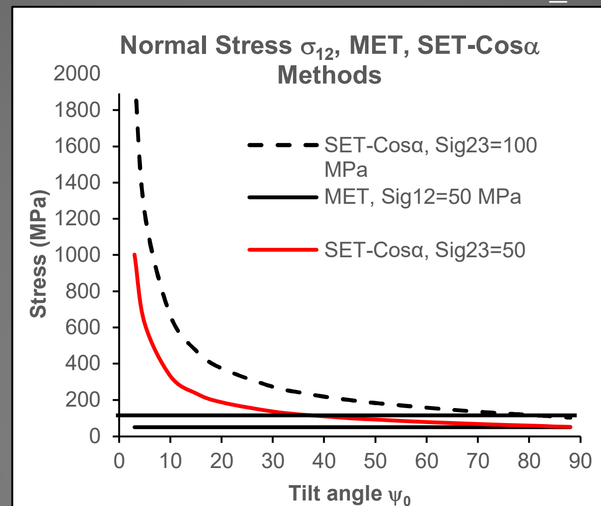
- Test under loading using 4-point bend specimen fabricated and tested according to ASTM E-1426. For this carbon steel: XEC = 5.14.10^{-6} MPa^{-1}, Bragg angle 2Θ=156°, E= 210 GPa.
- Measured stress tensor on the 4-point bend specimen prior to loading:

$$\sigma_{ij}(MPa) = \begin{pmatrix} -4\pm10 & +6\pm7 & -48\pm2 \\ +6\pm7 & -258\pm10 & -5\pm2 \\ -48\pm10 & -5\pm2 & 0\pm6 \end{pmatrix}$$

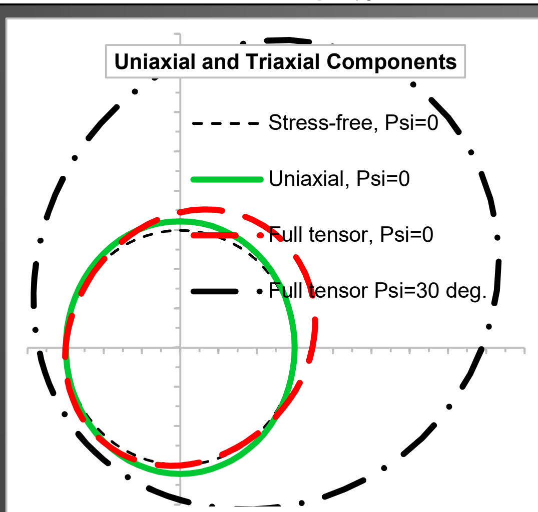
3. Experiment
- Residual stress measurements on: Aluminum, Stellite and Stainless Steel materials.

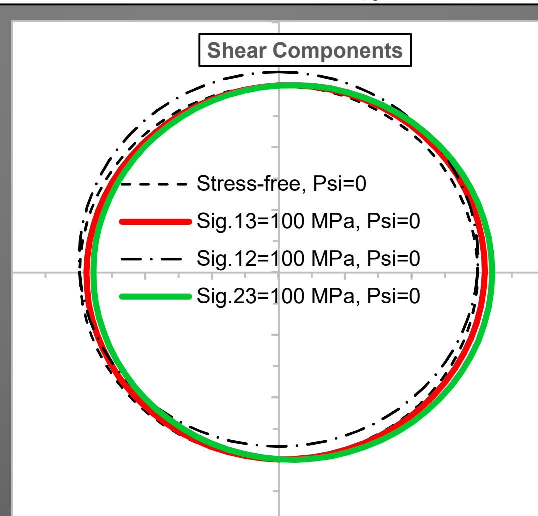
Simulation of SET- Sin²ψ, MET- Sin²ψ SET-Cosα with ψ_0 inclination angle





Presence of shear stress, change of σ_{11} and σ_{12} with ψ_0 tilt angle

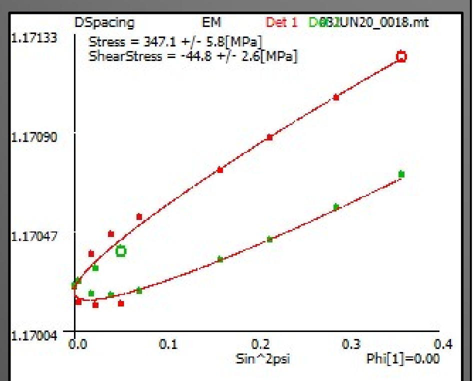


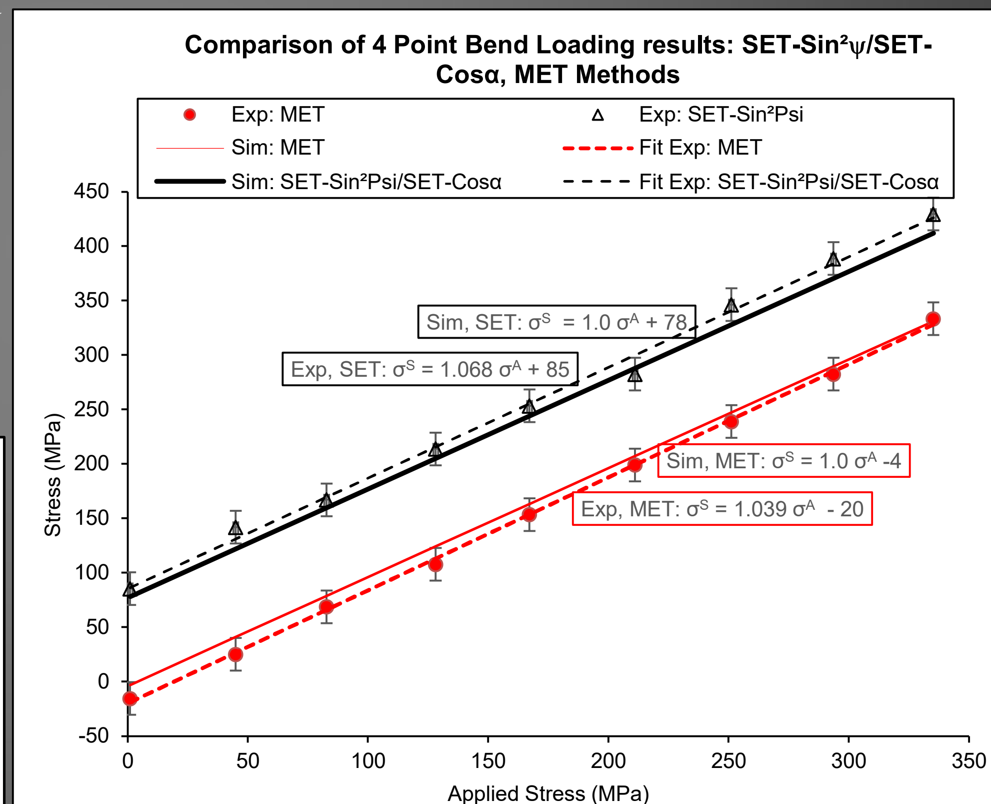


Debye ring distortion with stress tensor components

Simulation and experimental 4-point bend loading results SET-Cosα, SET- Sin²ψ MET- Sin²ψ

- Agreement between the measurements and the simulation for all techniques
- Difference in residual stress levels at zero load between SETs and MET.
- Offset of SET- $Sin^2\psi$ and SET-$Cos\alpha$, when compared to MET- $Sin^2\psi$, presence of shear stress σ_{13} in the material.
- All slopes are within the experimental error close to 1.



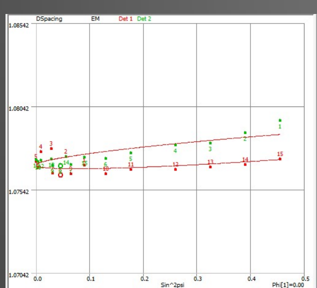


Results:

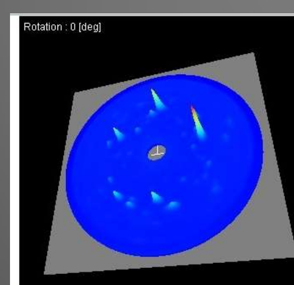
Measured stresses on different materials

Goniometer Geometry/Mode	Technique		Stress (MPa) Aluminum	Stress (MPa) Stellite	Stress (MPa) Stainless Steel
Omega	$Sin^2\psi$	MET	Normal stress: -18±19 Shear stress:-4±9	Normal stress: +436±14 Shear stress: +99±6	Normal stress: +124±90 Shear stress: +30±44
		SET	Normal stress: +7	Normal stress: +138	Normal stress: -46
Debye Ring	Cosα	SET	Failed	Normal stress: +3421±3702 [F]	Normal stress: +4574±2206 [F]

MET-$Sin^2\psi$

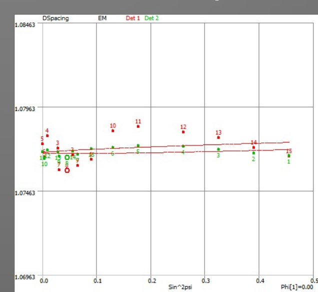


SET-Cosα

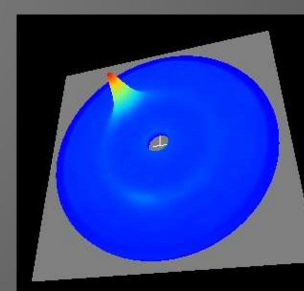


Material: Satellite

MET-$Sin^2\psi$



SET-Cosα



Material: Stainless steel

The difference is in the stress model used in the software (elliptical fit, multi-linear fit for triaxial measurement) and the tilt angles distributed over the half-space.

Testing	SET- $Sin^2\psi$	SET-Cosα	MET- $Sin^2\psi$
Space covered	¼-Space	¼-Space	½-Space
Free stress powder	Yes	Yes	Yes
Shot peened high stress standard	Yes	Yes	Yes
Uniaxial loading	Yes	Yes	Yes
High stress standard with shear stress	No	No	Yes
Heterogenous materials	No	No	Yes

Other issues we can mention with SET-Cosα are:

- Stress gradient,
- Surface roughness.
- Psi offset: surface inclination, curved surface

Residual Stress Standards and Guidelines

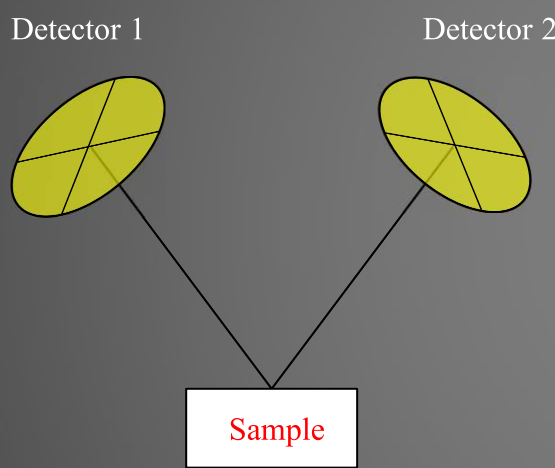
1. ASTM E-915 – (2010) “Standard test method for verifying the alignment of x-ray diffraction instruments for residual stress measurement”.
   - Normal and Shear Stresses are required.

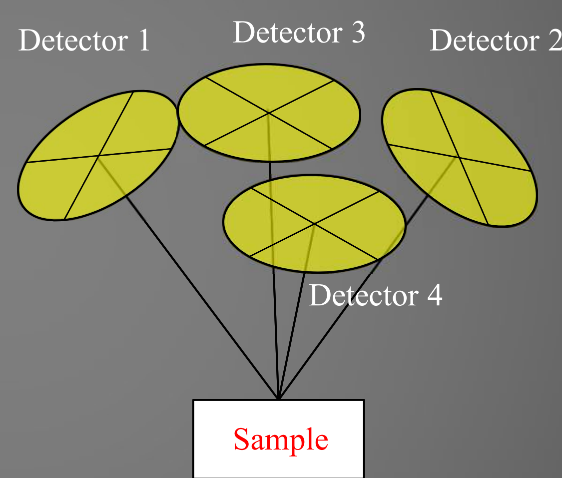
2. ASTM E2860 (2012):” Standard Test Method for Residual Stress Measurement by X-Ray Diffraction for Bearing Steels”.
   - MET is recommended.
   - Normal and Shear Stresses are required.
   - Comprehensive procedure for residual stress measurement on bearings

3. EN 15305 (2008): “Non-destructive testing - Test method for residual stress analysis by X-ray diffraction“.
   - MET is recommended.
   - Normal and Shear Stresses are required.
   - Detailed procedure for stress measurements for general conditions.
4. ASM Handbook, Vol. 11 (2020)
5. SAE HS-784 (2003)
6. SEM Handbook, (1996)
7. Others…

Alternative solutions:

1. Use 2 Debye rings for measurement in a single direction.
2. Use multiple Debye-rings for triaxial measurement





$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \cancel{\sigma_{22}} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \cancel{\sigma_{33}} \end{pmatrix}$$

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix} \text{ if } D_0 \text{ is known.}$$

Conclusions

1. SET-Sin$^2\psi$ is similar to SET-Cosα, equal stress values.
2. SET-Sin$^2\psi$, SET-Cosα cannot measure shear stress component σ_{13}, necessary for instrument alignment, surface orientation and curvature detection.
3. Normal stresses measured with SET-Sin$^2\psi$ and SET-Cosα are affected by the presence of σ_{13}. Same is true for σ_{12} which is affected by σ_{23} in case of SET-Cosα.
4. Reliable measurements must include both ψ^+ and ψ^-, such as MET to cover ½-space.
5. SET-Sin$^2\psi$ was not a reliable technique and it was not considered in any standard. SET-Cosα is similar.
6. No assumptions should be made prior to stress measurement. Limited results (σ_{11}) can be detrimental for decision making in the industry.
7. Cosα technique can be developed with multiple Debye rings to cover both ψ^+ and ψ^-, equivalent to MET-Sin$^2\psi$.
8. Existing tools for evaluation of instruments must include the stress model used in the software.
9. Standards should evolve to include triaxial stress state and shear components for testing and calculation models to guarantee the best outcome and most reliable results.